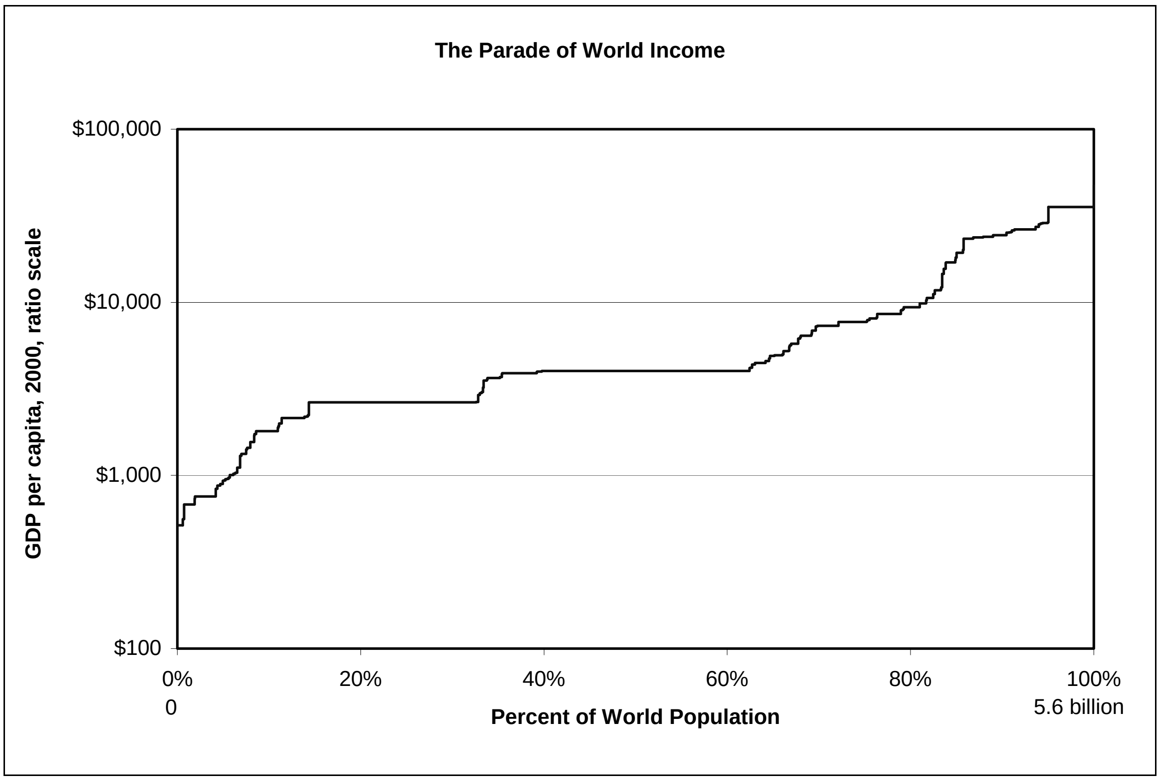
Chapter 1

The Facts to Be Explained

Solutions to Problems

1. A ratio scale transforms absolute differences in the variable of interest to proportional differences. For instance, the GDP of country *X*, whose GDP is 10 times greater than country *Y*, will be the same distance apart as a country *Z* whose GDP is 10 times smaller than country *Y*'s GDP, ie. the distance between *X*, *Y*, and *Z* will be the same. On a common linear scale, the distance between *X* and *Y* would be 10 times greater than the distance between *Y* and *Z*. As a result, transforming Figure 1.1 into a ratio scale will convey the absolute differences in the height of marchers into proportional differences.



2. Using the rule of 72, we know that GDP per capita will double every 72/2 years, ie. every 36 years. Therefore, if in year 0, GDP per capita is *x*, in year 36, GDP per capita will be 2*x*. Continuing with the same logic, in year 36 + 36 (= year 72), GDP per capita will be 4*x*, and in year 36 + 36 + 36 (= year 108), GDP per capita will be equal to 8*x*. We now arrive at our solution: It will take approximately 108 years for GDP per capita to increase by a factor of eight.

3. Using the rule of 72, we know that GDP per capita will double every 72/g years, where g is the annual growth rate of GDP per capita. Working backwards, if we start in the year 1900 with a GDP per capita of $1,000, to reach $4,000 by the year 1948, GDP per capita must have doubled twice. To see this, note that after doubling once, GDP per capita would be $2,000 in some year, and doubling again, GDP per capita would be $4,000, exactly the GDP per capita in year 1948. Using the fact that GDP doubled twice within 48 years and assuming a constant annual growth rate, we conclude that GDP per capita doubles every 24 years. Solving for the equation, $72 / g = 24$, we get g, the annual growth rate, to be 3% per year.

4. Between-country inequality is the inequality associated with average incomes of different countries. Country *A*'s average income is given by adding Alfred's Income and Doris's Income and then dividing by 2. This yields an average income of 2,500 for Country *A*. Similar calculations reveal that Country *B*'s average income is 2,500. Because the average income for Country *A* is equal to that of Country *B*, there is no between-country inequality in this world.

Within-country inequality is the inequality associated with incomes of people in the same country. In Country *A*, Alfred earns 1,00 while Doris earns 4,000, making it an income disparity of 3,000. In Country *B*, the income disparity is 1,000. Therefore, we see within-country income inequality in both Country *A* and Country *B*.
Because there is no between-country inequality, world inequality can be entirely attributed to within-country inequality.

Equivalently, one could calculate the mean log deviation to attain values for within- and between-country inequality. Using the formula on page 19, the value for between-country inequality is 0 whereas, the value for within-country inequality for Country *A* and Country *B* is 0.223 and 0.020, respectively. This implies the same conclusion as before.

5. We can solve for the average annual growth rate, g, by substituting the appropriate values into the equation:

$$(Y_{1900}) * (1+g)^{100} = Y_{2000} .$$

Letting $Y_{1900} = \$1,433$, $Y_{2000} = \$26,375$, and rearranging to solve for g, we get :

$$g = (\$ 26,375 / \$ 1,433)^{(1/100)} - 1,$$

$$g \approx 0.0296.$$

Converting g into a percent, we conclude that the growth rate of income per capita in Japan over this period was approximately 2.96% per year.

To find the income per capita of Japan 100 years from now in 2100, we solve

$$(Y_{2000}) * (1+g)^{100} = Y_{2100} .$$

Letting Y_{2000} = $26,375 and $g = 0.0296$,

$$(\$\,26{,}375) * (1+0.0296)^{100} = Y_{2100}\,,$$

$$Y_{2100} = \$\,485{,}443.60.$$

That is, if Japan grew at the average growth rate of 2.96% per year, we would find the income per capita of Japan in 2100 to be about $485,443.60.

6. In order to calculate the year in which income per capita in the United States was equal to income per capita in Sri Lanka, we need to find t, the number of years that passed between the year 2000 and the year US income per capita equaled that of 2000 Sri Lanka income per capita. Equating income per capita of Sri Lanka in year 2000 to income per capita of the United States in year 2000–t, we now write an equation for the United States as

$$(Y_{US,\,2000\text{-}t}) * (1+g)^{t} = Y_{US,\,2000}\,.$$

Since $Y_{US,\,2000\text{-}t} = Y_{Sri\ Lanka,\,2000}$ = $3,527 , $Y_{US,\,2000}$ = $35,587 , and $g = 0.019$, we then substitute in these values and solve for t.

$$(\$3{,}527) * (1+0.019)^{t} = \$35{,}587.$$

$$(1+0.019)^{t} = (\,\$35{,}587\,/\$3{,}527)$$

Taking the Natural Log of both sides, and noting that $ln(x^{y}) = y\ ln\ (x)$, we get

$$t\ \ ln\ (1+0.019) = ln\ (\,\$35{,}587\,/\$3{,}527)$$

$$t = 122.81$$

That is, 122.81 years ago, the income per capita of the United States equaled that of Sri Lanka's income in the year 2000. This year was roughly 2000–t, ie. the year 1877.

Solutions to Appendix Problems

1. (a) The level of GDP per capita in each country, measured in its own currency is

(CPUs per capita* Price) + (IC per capita* Price) = GDP per capita.

Therefore, Richland's GDP per capita is 40 and Poorland's GDP per capita is 4.

(b) The market exchange rate is determined by the law of one price. As CPUs are the only traded good, the price of computers should be the same. Consequently, the exchange rate must be 2 Richland dollars to 1 Poorland dollar.

(c) To find the ratio of GDP per capita between Richland and Poorland, we must first convert GDP denominations into the same currency. In the analysis that follows, I choose to convert GDP denominations into Poorland dollars, but converting to Richland dollars is equally correct, similar, and will yield the same result. From part (a), we convert Richland's GDP per capita, denominated in Richland dollars, into Poorland Dollars by multiplying GDP per capita with the market exchange rate. Since from part (b), we know 2 Richland Dollars equals 1 Poorland Dollar, we multiply 1/2 to Richland's GDP per capita, yielding 20 Poorland Dollars. Thus, the ratio of Richland GDP per capita to Poorland GDP per capita is 5:1.

(d) A natural basket to use is 3 computers and 1 ice cream. The cost of this basket in Richland is 10 Richland dollars. The cost of this basket in Poorland is 4 Poorland dollars. Equating the costs of baskets to be one price, the purchasing power parity exchange rate must be, 10 Richland Dollars : 4 Poorland Dollars

(e) To find the ratio of GDP per capita between Richland and Poorland, we must first convert GDP denominations into the same currency. In the analysis that follows, I choose to convert GDP denominations into Poorland dollars, but converting to Richland dollars is equally correct, similar, and will yield the same result. From part (a), we convert Richland's GDP per capita, denominated in Richland dollars, into Poorland Dollars by multiplying GDP per capita with the PPP exchange rate. Since from part (d), we know 10 Richland Dollars equals 4 Poorland Dollars, we multiply 4/10 to Richland's GDP per capita, yielding 16 Poorland Dollars. Thus the ratio of Richland GDP per capita to Poorland GDP per capita is 4:1.